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A family of $\mathcal{N} = 1$ SU$(N)^k$ theories from branes at singularities

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Abstract: We obtain $\mathcal{N} = 1$ SU$(N)^k$ gauge theories with bifundamental matter and a quartic superpotential as the low energy theory on D3-branes at singular points. These theories generalize that on D3-branes at a conifold point, studied recently by Klebanov and Witten. For $k = 3$ the defining equation of the singular point is that of an isolated D_4 singularity. For $k > 3$ we obtain a family of multimodular singularities. The considered SU$(N)^k$ theories flow in the infrared to a non-trivial fixed point. We analyze the AdS/CFT correspondence for our examples.

Keywords: Duality in Gauge Field Theories, Superstring Vacua, Brane Dynamics in Gauge Theories.

JHEP02(1999)019

Contents

1. Introduction

Dirichlet-branes [2] have proved to be an extremely useful tool for the study of gauge theories. The low-energy theory of N coincident D-branes placed at a regular point of the transversal space is maximally supersymmetric $\mathrm{U}(N)$ Yang-Mills. More general theories with less supersymmetry can be obtained by placing D-branes at singular points of the transversal space. Particular examples that have been extensively studied in the literature are D-branes at orbifold points [3, 4, 5].

In [6] Maldacena proposed a very interesting duality between large N gauge theories and type IIB or M-theory in a background given by the near horizon geometry of black-branes. The near horizon geometry of 3-branes at a regular point of the transverse space is $AdS_5 \times S^5$. The conjectured duality proposes that $\mathcal{N} = 4$ $\mathrm{SU}(N)$ Yang-Mills theory is dual to type IIB string theory on $AdS_5 \times S^5$, where S^5 bears N units of five-form flux. There has been a big effort in extending this duality to the case of 3-branes sitting at a singular point of the transverse space. In that case S^5 is substituted by a five-dimensional manifold H, which describes the angular part of the singular space. When the singular space has an orbifold description, H is given by S^5/Γ where Γ is the discrete orbifold group [7]. For $\Gamma \subset \mathrm{SU}(2)$, $\mathrm{SU}(3)$ or $\mathrm{SU}(4)$ the dual gauge theories have $\mathcal{N} = 2, 1$ or 0 conformal symmetry respectively.

Klebanov and Witten considered D3-branes at a conifold point in [8]. The associated gauge theory is $\mathcal{N} = 1$ $\mathrm{SU}(N) \times \mathrm{SU}(N)$ (plus a free $\mathrm{U}(1)$) with bifundamental matter multiplets and a quartic superpotential. The AdS/CFT correspondence for this case has been analyzed in great detail [8, 9, 10]. Branes at spaces with more general singularities have been recently considered in [11, 12, 13], where the associated gauge theories were also derived. Our aim in this paper is to find a class of singular spaces such that the world-volume theory on D3-branes at the singular point generalizes that of [8] to the case of k factor groups. In order to achieve this goal, we will follow a somehow opposite approach to that used in [8, 11, 12, 13]. Instead of deriving the field theory once the singular space is known, we will use the field theory data to construct the singular three-fold. Section 2 will be devoted to studying the case $k = 3$. We will find that the associated singular space describes an isolated D_4-singularity of a three-fold.

Our $\mathrm{SU}(N)^k$ theories flow in the infrared to a non-trivial fixed point. In section 3 we will analyze the AdS/CFT correspondence for the case $k = 3$. We will see that making certain assumptions about the topology of the D_4-singularity, we obtain a consistent AdS/CFT correspondence. In particular, we will compare moduli spaces of gauge and string theory, global symmetries of the gauge theory with gauge symmetries of string dual and the spectrum of baryonic operators with branes wrapped on homology cycles of H [10, 14]. In section 4 we generalize our results to $k > 3$.

After this paper was completed, we received reference [15], where the same problem is treated.

2. Branes at singularities

The gauge theory on N parallel D3-branes at a conifold point is $\mathcal{N} = 1$ $\mathrm{U}(N) \times \mathrm{U}(N)$ with four chiral multiplets A_i, B_j, $i, j = 1, 2$, transforming in the $(\mathbf{N}, \bar{\mathbf{N}})$ and $(\bar{\mathbf{N}}, \mathbf{N})$ representations of the gauge group respectively [8]. We wish to find a singular space such that the gauge theory on D3-branes at the singular point is the generalization of the previous one to the case of three factor groups, i.e. $\mathcal{N} = 1$ $\mathrm{U}(N) \times \mathrm{U}(N) \times \mathrm{U}(N)$ with chiral matter fields

$$\begin{aligned} A &= (\mathbf{N}, \bar{\mathbf{N}}, 1)\,, \qquad & B &= (1, \mathbf{N}, \bar{\mathbf{N}})\,, \qquad & C &= (\bar{\mathbf{N}}, 1, \mathbf{N})\,, \\ \tilde{A} &= (\bar{\mathbf{N}}, \mathbf{N}, 1)\,, \qquad & \tilde{B} &= (1, \bar{\mathbf{N}}, \mathbf{N})\,, \qquad & \tilde{C} &= (\mathbf{N}, 1, \bar{\mathbf{N}})\,. \end{aligned} \tag{2.1}$$

The superpotential of the gauge theory will be determined by the criterion that a branch of the moduli space can be interpreted as positions of D3-branes.

It is convenient to consider first the case $N = 1$. This should correspond to a single D3-brane in our searched for singular space. The D-term equations are

$$\begin{aligned} |A|^2 + |\tilde{B}|^2 - |\tilde{A}|^2 - |B|^2 &= \xi_1\,, \\ |B|^2 + |\tilde{C}|^2 - |\tilde{B}|^2 - |C|^2 &= \xi_2\,, \\ |C|^2 + |\tilde{A}|^2 - |\tilde{C}|^2 - |A|^2 &= \xi_3\,. \end{aligned} \tag{2.2}$$

The real numbers ξ_i are Fayet-Iliopoulos parameters. Since the sum of the lhs's in (2.2) is zero, the parameters ξ_i must satisfy $\sum \xi_i = 0$ in order to allow for a supersymmetric vacuum. The fact that only two of the three D-term equations are linearly independent implies that there is a combination of U(1)'s under which all the fields are uncharged. This U(1) field will be thus free and decouple. We will denote by Q the space of solutions to (2.2) quotiented by the gauge transformations generated by the two non-trivial U(1)'s. The space Q has complex dimension four.

We can describe Q directly in terms of gauge invariant quantities [16]. A minimal set of gauge invariant quantities is

$$x_1 = A\tilde{A}\,, \qquad x_2 = B\tilde{B}\,, \qquad x_3 = C\tilde{C}\,, \qquad z = ABC\,, \qquad w = \tilde{A}\tilde{B}\tilde{C}\,. \quad (2.3)$$

These variables are not independent, they are subject to the constraint

$$x_1 x_2 x_3 = zw\,. \quad (2.4)$$

This equation defines a hypersurface in $\mathbb{C}^5$. The space (2.4) is singular along the following codimension three subspaces:

$$\begin{array}{ll} i) & x_1 = x_2 = y = z = 0\,; \\ ii) & x_1 = x_3 = y = z = 0\,; \\ iii) & x_2 = x_3 = y = z = 0\,. \end{array}$$

The three lines of singularities intersect at the origin $x_i = y = z = 0$. When $\xi_i = 0$, Q coincides with the four-fold (2.4). Values $\xi_i \neq 0$ (for some i) correspond to (partial) resolutions of (2.4) in which a singular subspace is substituted by $\Sigma \times \mathbb{C}$, where Σ denotes a two-sphere.

The moduli space of the gauge theory is the subspace of Q determined by the F-term equations. Let us consider the quartic superpotential

$$W = (a_1 x_1 + a_2 x_2 + a_3 x_3)^2\,, \quad (2.5)$$

where a_i are complex parameters. The F-term equations derived from W reduce to a single relation expressible in terms of the variables x_i

$$a_1 x_1 + a_2 x_2 + a_3 x_3 = 0\,. \quad (2.6)$$

The moduli space of the gauge theory is given by the intersection between Q and the hyperplane (2.6) in $\mathbb{C}^5$. This defines a six-dimensional space, thus susceptible of being interpreted as the transversal space to a D3-brane. The quartic superpotential in (2.5) is the most general one with this property. For $a_i \neq 0$ the only singular point contained in the intersection between Q and (2.6) is the origin. We can use the hyperplane equation to eliminate one of the x_i variables in (2.4). When $a_i \neq 0$, we obtain

$$-\left(\frac{a_1}{a_2}x_1 + \frac{a_3}{a_2}x_3\right) x_1 x_3 = zw\,. \quad (2.7)$$

We will denote this space by K. It is indeed only singular at the origin. By an obvious linear change of coordinates we can rewrite it as

$$x^3 + y^2 x = zw\,, \quad (2.8)$$

which is the standard form of a D_4 singularity of a complex three-fold [17]. Since the parameters a_i can be eliminated by a coordinate change, they do not affect the complex structure of K.

The superpotential (2.5) can be extended to that of an $\mathrm{U}(N)^3$ theory with matter content (2.1)

$$\begin{aligned} W = a_1^2 \mathrm{Tr}\, \left(A\tilde{A}\right)^2 + a_2^2 \mathrm{Tr}\, \left(B\tilde{B}\right)^2 + a_3^2 \mathrm{Tr}\, \left(C\tilde{C}\right)^2 + 2a_1a_3 \mathrm{Tr}\, A\tilde{A}\tilde{C}C + \\ + 2a_1a_2 \mathrm{Tr}\, B\tilde{B}\tilde{A}A + 2a_2a_3 \mathrm{Tr}\, C\tilde{C}\tilde{B}B\,. \end{aligned} \tag{2.9}$$

All the matter fields are uncharged under the diagonal combination of the three U(1) factors in $\mathrm{U}(N)^3$, thus this U(1) is free. The other two U(1) fields have positive beta functions and are expected to decouple in the infrared limit.[1] Therefore from now on we will actually work with an $\mathrm{SU}(N)^3$ theory, instead of $\mathrm{U}(N)^3$. The F-term equations derived from (2.9) imply that (2.7) is still verified, but now by the matrix quantities

$$x_1 = A\tilde{A}\,, \qquad x_3 = \tilde{C}C\,, \qquad y = ABC\,, \qquad z = \tilde{C}\tilde{B}\tilde{A}\,. \tag{2.10}$$

Notice that all these quantities transform in the adjoint representation of the first $\mathrm{SU}(N)$ group factor (plus a singlet). Of course, analogous relations can be derived involving operators which transform in the adjoint of the second and the third $\mathrm{SU}(N)$ factors. A family of vacua solving the D- and F-term equations is given by matrices that are, in some basis, diagonal and such that each entry satisfies (2.6) and (2.7). Along these vacua the superpotential (2.9), plus the Higgs mechanism, will give masses to the non-diagonal excitations. This family of vacua reproduces N copies of the space K. In order to obtain the moduli space of the theory we should quotient by the Weyl transformations of $\mathrm{SU}(N)_i$. Since all the operators in (2.10) transform in the adjoint representation of $\mathrm{SU}(N)_1$, quotienting by Weyl transformations produces the moduli space K^N/S_N, where S_N is the group of permutations of N elements. This is precisely the moduli space associated to positions of N D3-branes in the space K.

In order that a certain singular complex manifold is a valid compactification space for string theory, the singularity must be of a restricted type known as Gorenstein canonical singularity [11]. This means that there exists a non-vanishing holomorphic top form near the singularity that extends to a holomorphic form on any smooth blow up of the singularity. The D_4 singularity is of this type [19, 1]. Therefore it is consistent to interpret the $\mathcal{N} = 1$ $\mathrm{SU}(N)^3$ gauge theory with matter content (2.1) and superpotential (2.9) as the low-energy theory of N D3-branes at the singular space K.

[1] For gauge theories on D3-branes at orbifold singularities the non-trivial U(1) fields have been shown to be spontaneously broken [3] or anomalous [18].

The $\mathrm{SU}(N)^3$ theory has baryon operators $B_i = X_i^N$, $\tilde{B}_i = \tilde{X}_i^N$ for $X_i = A, B, C$. A non-zero expectation value for one of the baryon operators higgses the original theory down to $\mathrm{SU}(N) \times \mathrm{SU}(N)$. Let us consider $\langle A \rangle \sim 1$ and substitute this in (2.9). The non-zero vev induces a mass term for $\tilde{A}$ and thus only $B, \tilde{B}$ and $C, \tilde{C}$ remain as light fields. Integrating out $\tilde{A}$ we obtain the superpotential

$$W = 2a_2 a_3 \mathrm{Tr}\,(C\tilde{C}\tilde{B}B - \tilde{C}CB\tilde{B})\,. \tag{2.11}$$

This is precisely the superpotential of the $\mathrm{SU}(N)^2$ theory obtained on N D3-branes at a conifold [8]. Along the baryonic branches, some of the ξ_i parameters in (2.2) are non-zero. We have argued that non-zero ξ_i correspond to resolutions or partial resolutions of the singular space transverse to the D3-branes. Thus, if our $\mathrm{SU}(N)^3$ theory lives on the world-volume of D3-branes at a D_4 singularity, it is necessary that this singularity can be partially resolved by a single blow up to a conifold.

It is convenient to use the standard form (2.8) of the D_4 singularity: $x^3 + y^2 x = zw$. We can partially resolve it by blowing up the space (x, y, z, w) at $x = z = 0$ as explained in [20]. The partially resolved surface is covered by open sets $(\tilde{x}, y, z, w) = (x, y, \tilde{z}, w)$, glued together by the conditions $\tilde{x}\tilde{z} = 1$ and $z = x\tilde{z}$. The inverse image of our surface in the first set is $w = z^2\tilde{x}^3 + y^2\tilde{x}$, which is non-singular. In the second set we obtain $x^2 + y^2 = \tilde{z}w$, which is the defining equation of the conifold. The inverse image of the singular point $x = y = z = w = 0$ is the $\mathbb{P}_1$ parameterized by $\tilde{x}$ in the first patch and $\tilde{z}$ in the second patch.

The group of non-anomalous continuous global symmetries of the $\mathrm{SU}(N)^3$ theory is $\mathrm{U}(1)_R \times \mathrm{U}(1)_1 \times \mathrm{U}(1)_2 \times \mathrm{U}(1)$, under which the fields transform as indicated in the table.

	$\mathrm{U}(1)_R$	$\mathrm{U}(1)_1$	$\mathrm{U}(1)_2$	$\mathrm{U}(1)$
A	$1/2$	1	-1	1
$\tilde{A}$	$1/2$	-1	1	-1
B	$1/2$	0	1	0
$\tilde{B}$	$1/2$	0	-1	0
C	$1/2$	-1	0	0
$\tilde{C}$	$1/2$	1	0	0

The $\mathrm{U}(1)_1$ and $\mathrm{U}(1)_2$ symmetries are associated with the two non-trivial $\mathrm{U}(1)$ factors of the $\mathrm{U}(N)^3$ gauge group living on the D3-branes. Although these $\mathrm{U}(1)$ gauge fields are expected to decouple in the infrared, they survive as global symmetries [11]. The superpotential (2.9) is also invariant under charge conjugation, $X_i \to \tilde{X}_i^t$, $\tilde{X}_i \to X_i^t$, with $X_i = A, B, C$. Extra discrete symmetries occur for particular values of the a_i parameters in the superpotential. We will discuss them in the next section.

An interesting limit of the theory is achieved when one of the parameters a_i goes to zero. To be definite, we consider $a_3 \to 0$. Sending a_3 to zero while keeping a_1, a_2 finite would imply that the fields C, $\tilde{C}$ do not appear in the superpotential. This situation cannot represent a theory derived from parallel D3-branes. There are baryonic directions in which the theory gets higgsed down to $\mathrm{SU}(N)$. This corresponds to move the N D3-branes away from the singular point while keeping

JHEP02(1999)019

them together. The world-volume theory of D3-branes at a regular point of the transverse space must flow in the infrared to $\mathcal{N}=4$ Yang-Mills. However if one of the a_i is zero, the superpotential along the mentioned vacua does not reproduce that of an $\mathcal{N}=4$ theory. In order to avoid this problem we can perform a double limit in which we send a_3 to zero and a_1, a_2 to infinity such that

$$a_1/a_2 = a\,, \qquad a_2 a_3 = b \tag{2.12}$$

are kept finite. We obtain then the superpotential

$$W = a_2^2 \mathrm{Tr}\, \left(B\tilde{B} + a\tilde{A}A\right)^2 + 2b\mathrm{Tr}\, \left(C\tilde{C}\tilde{B}B + a\tilde{C}CA\tilde{A}\right). \tag{2.13}$$

The first term on the right hand side becomes infinite in the limit that we are considering. Its form is such that it can be interpreted as a divergence due to integrating out a chiral field ϕ in the adjoint representation of the second $\mathrm{SU}(N)$, with mass $m \sim a_2^{-2}$. Integrating in this field we get

$$W = c\mathrm{Tr}\, \phi \left(B\tilde{B} + a\tilde{A}A\right) + 2b\mathrm{Tr}\, \left(C\tilde{C}\tilde{B}B + a\tilde{C}CA\tilde{A}\right). \tag{2.14}$$

This theory was recently analyzed in [11, 13]. There it was proposed to be the world-volume theory on D3-branes at the singular space

$$xy^2 = zw\,. \tag{2.15}$$

We observe that performing the previous double limit in (2.7) we obtain, up to rescalings, this same space. It corresponds to intersect Q with a hyperplane containing one of the four-fold singular subspaces. We will denote it by $\tilde{K}$. It is singular at the codimension two subspace $y = z = w = 0$.

The existence of a limit in which adjoint matter becomes massless has the following origin. Our $\mathcal{N}=1$ theory is related to an $\mathcal{N}=2$ theory with the same gauge group and matter content. Notice that the fields (2.1) can be paired up to form three $\mathcal{N}=2$ hypermultiplets. That $\mathcal{N}=2$ gauge theory can be derived from D3-branes at a $\mathbb{Z}_3$ orbifold, with $\mathbb{Z}_3$ acting only on four of the six transversal coordinates [3]. The associated superpotential is

$$W = \mathrm{Tr}\ \phi_1 \left(A\tilde{A} - \tilde{C}C\right) + \mathrm{Tr}\ \phi_2 \left(B\tilde{B} - \tilde{A}A\right) + \mathrm{Tr}\ \phi_3 \left(C\tilde{C} - \tilde{B}B\right), \tag{2.16}$$

where ϕ_i are chiral multiplets transforming in the adjoint of each gauge group factor. We can break $\mathcal{N}=2$ to $\mathcal{N}=1$ by giving masses to the adjoint fields

$$\Delta W = m_1 \mathrm{Tr}\ \phi_1^2 + m_2 \mathrm{Tr}\ \phi_2^2 + m_3 \mathrm{Tr}\ \phi_3^2\,. \tag{2.17}$$

Integrating out the adjoint fields we obtain the superpotential

$$W = -\frac{1}{4m_1}\mathrm{Tr}\ \left(A\tilde{A} - \tilde{C}C\right)^2 - \frac{1}{4m_2}\mathrm{Tr}\ \left(B\tilde{B} - \tilde{A}A\right)^2 - \frac{1}{4m_3}\mathrm{Tr}\ \left(C\tilde{C} - \tilde{B}B\right)^2. \tag{2.18}$$

This superpotential is of the form (2.9) when $\sum m_i = 0$. The condition $\sum m_i = 0$ implies that the mass perturbation (2.17) can be written as $m\mathrm{Tr}\ (\phi_1^2 - \phi_2^2) + m'\mathrm{Tr}\ (\phi_1^2 - \phi_3^2)$. Let us consider switching on m' but keeping $m = 0$. After integrating out ϕ_1 and ϕ_3, and redefining $\phi_2 \to \phi_2 - \frac{1}{4m'}(B\tilde{B} + \tilde{A}A)$, we obtain (2.14) with $c = -a = 1$, $b = 1/8m'$. The superpotential obtained by breaking $\mathcal{N} = 2$ to $\mathcal{N} = 1$ by adjoint masses contains less free parameters than (2.9) or (2.14). The reason is that $\mathcal{N} = 2$ supersymmetry fixes the coupling between adjoint fields and hypermultiplets. Once $\mathcal{N} = 2$ is broken new parameters can be introduced in the superpotential associated to these couplings, as it is clearly the case in (2.14). The only restriction we impose on the parameters is that the space of solutions to the D- and F-terms equations can be interpreted as positions of D3-branes in a six-dimensional transverse space. In the next section we will analyze which parameters survive in the infrared limit of the theory.

3. AdS dual theory

In a generic situation, the world-volume theory on D3-branes couples to the bulk degrees of freedom of type IIB string theory. We can decouple the gravity degrees of freedom and still obtain an interacting world-volume theory by sending the string tension to infinity ($\alpha' \to 0$) while keeping the string coupling fixed. This limit can also be performed in the supergravity metric representing a collection of N parallel black 3-branes. The resulting metric reproduces the near-horizon geometry of the black branes. In [6] Maldacena conjectured that, for large N, the gauge theory living on the D3-branes is dual to type IIB string theory on a background given by the near-horizon geometry of the black branes. In this section we want to apply this proposal to our example.

The supergravity metric representing N parallel 3-branes with flat world-volume placed at a certain point of a six-dimensional space is [21, 22]

$$ds^2 = f(y)^{-1/2} dx^2 + f(y)^{1/2} dy^2 \,, \tag{3.1}$$

where dx^2 is the flat four-dimensional Minkowski metric and dy^2 is the metric in the transversal space. The function $f(y)$ determines the expectation value of the 5-form field strength associated to the presence of the 3-branes and satisfies

$$\Delta f = -(2\pi)^4 N \frac{\delta^6(y - y_0)}{\sqrt{g}} \,, \qquad F_{0123i} = -\frac{1}{4}\partial_i f^{-1} \,, \tag{3.2}$$

with Δ denoting the Laplacian of the six-dimensional space, y_0 the point at which the N 3-branes sit, and 0123 being the directions along the world-volume of the 3-branes.

We are interested in the case that the transverse space is K, given by (2.7), and the 3-branes are placed at its singular point. Especially relevant is the case in which the metric on the transverse space, close to a singular point, can be written as [8, 23, 11]

$$dy^2 = r^2 \left(\frac{dr^2}{r^2} + d\Omega_H^2 + O(r/\alpha'^{1/2}) \right), \tag{3.3}$$

where r measures distances with respect to the singular point. H is a five-dimensional space given by points at a distance $r = 1$ from the singular point, also denoted horizon manifold of the singularity [11]. $d\Omega_H^2$ is a dimensionless metric on H, independent of r and α'. Expression (3.3) means that close to the singular point, the transverse space can be approximated by a cone over H. When (3.3) holds, the metric (3.1)–(3.2) turns in the decoupling of gravity limit into that of $AdS_5 \times H$ [6, 8]. We will assume that K admits a Ricci-flat metric that close to the singular point verifies (3.3).

For the case of zero superpotential each sector of the $\mathrm{SU}(N)^3$ gauge theory provides an $\mathcal{N} = 1$ $\mathrm{SU}(N)$ theory with $2N$ fundamental hypermultiplets. Although this theory has non-zero beta function, it is expected to flow in the infrared to a non-trivial fixed point [24]. The superpotential (2.9) is non-renormalizable as a peturbation of the free field theory, but it is a marginal peturbation of the superconformal theory [8]. We follow [8] in interpreting the AdS/CFT correspondence for our case. We propose that type IIB string theory on $AdS_5 \times H$, with H being the horizon manifold of a D_4 singularity, is dual to the superconformal field theory obtained by letting flow to the infrared fixed point a large N $\mathcal{N} = 1$ $\mathrm{SU}(N)^3$ gauge theory with matter content (2.1) and then perturbing it with the superpotential (2.9).

We will test this duality by comparing moduli spaces in both theories, finding the string counterpart of the global symmetries of the gauge theory and by matching baryon operators of the gauge theory with wrapped branes on H. For this we need to know the homology of H. We have seen in the previous section that the space K can be obtained, when $a_i \neq 0$, from the intersection between the four-fold Q and the hyperplane (2.6). The four-fold Q has three codimension three subspaces of singularities. When $a_i \neq 0$ the only singular point contained in the hyperplane is the origin of Q. The space K has thus an isolated singularity at the origin and therefore its horizon manifold H will be a smooth five-dimensional space. However when one of the constants a_i is zero, the hyperplane (2.6) intersects one of the singular subspaces of Q. The space we obtain is then $\tilde{K}$, with defining equation (2.15). This space was analyzed in detail in [11]. It contains a complex line of $\mathbb{Z}_2$ singularities. As a result of this, its horizon manifold $\tilde{H}$ is singular along an S^1. The space $\tilde{H}$ admits a description as $(S^3 \times S^3)/\mathrm{U}(1)$, similar to the conifold.[2] It has a non-trivial two-cycle and a non-trivial three-cycle. The transversal space to the singular S^1 in $\tilde{H}$ can be

[2]However contrary to the conifold the U(1) action is not regular, i.e. the orbits of the U(1) action do not have constant length. This gives rise to the circle of singularities.

described locally as $\mathbb{C}^2/\mathbb{Z}^2$, and resolved by replacing the singular point by a two-sphere Σ. This process generates an additional three-cycle in $\tilde{H}$ given by $S^1 \times \Sigma$, where S^1 is the circle of singularities.

Since both $\tilde{K}$ and K can be obtained from the four-fold Q by just continuously varying the orientation of the hyperplane (2.6), and the horizon manifold H of K is non-singular, we could interpret H as a deformation of $\tilde{H}$ that smoothes out the singularities. Based on this heuristic argument, we will assume the simplest situation that the second and third Betti numbers of H are $b_2 = b_3 = 2$. Support for this hypothesis will come from obtaining a consistent AdS/CFT correspondence. In this way the duality proposed by Maldacena can also be used to learn about the topology of singular spaces [25], which is in many cases an open problem.

As a first check, we compare the moduli space of string and superconformal field theory. With the previous hypothesis for the homology groups of H, type IIB string theory on $AdS_5 \times H$ possesses a (complex) three dimensional moduli space. The moduli parameters are the complexified string coupling constant and two additional complex parameters coming from integrating the B fields over the two homology two-cycles.

The presence of truly marginal deformations of the conformal theory can be analyzed with the methods of [26]. The scale dependence of the gauge couplings and coupling constants appearing in the superpotential is given in terms of the anomalous dimensions of the fields by

$$\begin{aligned} A_{g_1} &= A_{h_{AC}} = 1 + \gamma_A + \gamma_C \,, & A_{h_{A^2}} &= 1 + 2\gamma_A \,, \\ A_{g_2} &= A_{h_{BA}} = 1 + \gamma_B + \gamma_A \,, & A_{h_{B^2}} &= 1 + 2\gamma_B \,, \\ A_{g_3} &= A_{h_{CB}} = 1 + \gamma_C + \gamma_B \,, & A_{h_{C^2}} &= 1 + 2\gamma_C \,. \end{aligned} \tag{3.4}$$

We have used $\gamma_{X_i} = \gamma_{\tilde{X}_i}$, $X_i = A, B, C$, because charge conjugation is a symmetry of the theory. The vanishing of all (3.4) only imposes three relations. Since we want to preserve the geometrical interpretation of the gauge theory moduli space, it is crucial that the superpotential contains only three independent coupling constants, as in (2.9). We have then six couplings and three relations they must satisfy. The conformal theory we are considering will thus have three marginal couplings, as its proposed string dual.

We now turn to obtain the string counterpart to the global symmetries of the gauge theory. It will be convenient to use the description of K in terms of the variables (x_1, x_3, z, w), since they have a direct gauge theory interpretation. The space K has a scaling symmetry $(x_1, x_3, z, w) \to (\lambda^2 x_1, \lambda^2 x_3, \lambda^3 z, \lambda^3 w)$, with $\lambda \in \mathbb{C}^*$. Restricting $\lambda = e^{i\alpha}$ we obtain a set of U(1) transformations that act as $\Omega \to e^{4i\alpha}\Omega$ on the holomorphic top form

$$\Omega = \frac{dx_1 \wedge dx_3 \wedge dz}{z} \,. \tag{3.5}$$

This is thus an R-symmetry in type IIB string theory[3] which we can match with the non-anomalous $\mathrm{U}(1)_R$ symmetry of the field theory [8]. The space K is also invariant under the transformation $z \to e^{i\beta}z$, $w \to e^{-i\beta}w$. Since the top form is invariant under this transformation, it will not be an R-symmetry. We can associate it with the last U(1) in table 2. We have argued that $b_3(H) = 2$. Integrating the self-dual four-form of type IIB theory on the two non-trivial three-cycles we get two U(1) fields on AdS_5. According to [27, 28], gauge fields in AdS couple to global symmetry currents on the boundary. In this way the existence of two three-cycles implies two global U(1) symmetries in the field theory. We can identify them with $\mathrm{U}(1)_1$ and $\mathrm{U}(1)_2$ in table 2.

For generic a_i parameters in the superpotential the only discrete symmetry of the gauge theory is charge conjugation. However when some of the a_i are equal there are additional discrete symmetries. Let us consider $a_1 = a_3$. In this case, the superpotential is invariant under the interchange of the second and third $\mathrm{SU}(N)$ gauge factor together with the following action on the matter fields

$$A \leftrightarrow \tilde{C}\,, \qquad \tilde{A} \leftrightarrow C\,, \qquad B \leftrightarrow \tilde{B}\,. \tag{3.6}$$

We denote this transformation by $\mathcal{P}_{23}$. In terms of the string dual it corresponds to interchanging the internal space coordinates $x_1 \leftrightarrow x_3$ and $z \leftrightarrow w$. The top form (3.5) remains invariant under this operation in accordance with the fact that $\mathcal{P}_{23}$ is not an R-symmetry of the gauge theory. The composition of charge conjugation with $\mathcal{P}_{23}$ is also a symmetry of the gauge theory. Under it the second and third gauge factor are interchanged and the matter fields transform as

$$\begin{aligned} &A \to C^t\,, \qquad &&C \to A^t\,, \qquad &&B \to B^t\,, \\ &\tilde{A} \to \tilde{C}^t\,, \qquad &&\tilde{C} \to \tilde{A}^t, \qquad &&\tilde{B} \to \tilde{B}^t\,. \end{aligned} \tag{3.7}$$

In the dual string theory this transformation should correspond to acting with ω, the center of $\mathrm{SL}(2,\mathbb{Z})$, and interchanging $x_1 \leftrightarrow x_3$.

String theory compactified on K, when $a_1 = a_3$, has three $\mathbb{Z}_2$ symmetries:

1. the interchange of the x_1 and x_3 coordinates in the defining equation of K;

2. the interchange of z and w;

3. the center of $\mathrm{SL}(2,\mathbb{Z})$ of type IIB string theory, ω.

We have found that the composition of any two of these symmetries corresponds to a discrete symmetry of the associated field theory. We would like to propose that ω corresponds in field theory terms to the interchange of the second and third gauge

[3]We are assuming again that K is a non-compact Calabi-Yau three-fold. Then $\Omega = \eta\Gamma\eta^T$, with η a covariantly constant spinor. Thus transformations that act on Ω are R-symmetries.

factor and the action on the matter fields $X_i \to X_i^t$, $\tilde{X}_i \to \tilde{X}_i^t$, with $X_i = A, B, C$. It is important to notice that this transformation maps the $\mathrm{SU}(N)^3$ gauge theory with matter content (2.1) to an equivalent theory where the matter fields transform as

$$\begin{aligned} A &= (\bar{\mathbf{N}}, 1, \mathbf{N})\,, \qquad B = (1, \mathbf{N}, \bar{\mathbf{N}})\,, \qquad C = (\mathbf{N}, \bar{\mathbf{N}}, 1)\,, \\ \tilde{A} &= (\mathbf{N}, 1, \bar{\mathbf{N}})\,, \qquad \tilde{B} = (1, \bar{\mathbf{N}}, \mathbf{N})\,, \qquad \tilde{C} = (\bar{\mathbf{N}}, \mathbf{N}, 1)\,. \end{aligned} \tag{3.8}$$

Support for this interpretation is the following. In [13, 29], D3-branes at a conifold point were mapped into a brane configuration in type IIA, or M-theory, by using T-duality. The dual configuration consists of NS5-branes expanding along 012345 and 012389 directions and D4-branes expanding along 01236 with the x_6 coordinate living on a circle. In [29] it was proposed that ω of type IIB corresponds, in the dual M-theory set-up, to the transformation $x_6 \to -x_6$, $x_{10} \to -x_{10}$.

We saw in the previous section that our $\mathcal{N} = 1$ gauge theory can be derived from an $\mathcal{N} = 2$ theory with the same matter content by integrating out the adjoint fields. The $\mathcal{N} = 2$ theory can be easily dualized to an elliptic model [30] with three parallel NS5-branes (see figure 1). Let us associate $x_6 = 0$ to the position of the NS5-brane originating the fields B and $\tilde{B}$. The transformation $x_6 \to -x_6$, $x_{10} \to -x_{10}$ has the same effect as we have proposed for ω: it interchanges the second and third gauge factor without interchanging the matter fields. This brings the initial $\mathcal{N} = 2$ theory to an equivalent theory with matter content precisely as in (3.8). The action of $x_1 \leftrightarrow x_3$, and $z \leftrightarrow w$ on the field theory can then be deduced from ω and (3.6), (3.7).

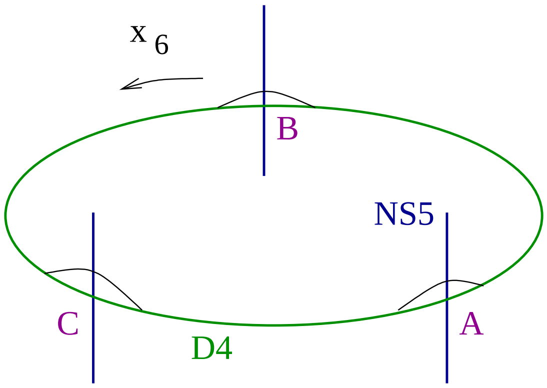

Figure 1: Type IIA brane configuration dual to D3-branes at a $\mathbb{C}^2/\mathbb{Z}_3$ orbifold.

Finally, we would like to match baryons of the gauge theory with D3-branes wrapped on three-cycles of the horizon manifold H [10, 14]. We have conjectured that H has two non-trivial three-cycles. The integration of the self-dual four-form of type IIB on these cycles gives two U(1) gauge fields in AdS_5. We have argued that they induce on the boundary field theory the global $\mathrm{U}(1)_1$ and $\mathrm{U}(1)_2$ symmetries of table 2. Let us denote by C_1 and C_2 the three-cycles associated to $\mathrm{U}(1)_1$ and $\mathrm{U}(1)_2$ respectively. D3-branes wrapped around C_i will correspond in the field theory to baryons charged under $\mathrm{U}(1)_i$, $i = 1, 2$. Therefore we can read from the quantum numbers of the matter fields how to relate wrapped D3-branes with baryon operators

$$B_A \to C_1 - C_2\,, \qquad B_B \to C_2\,, \qquad B_C \to -C_1\,. \tag{3.9}$$

The same relation holds for the antibaryons $B_{\tilde{X}}$, when instead of D3-branes we wrap anti D3-branes.

When one of the a_i parameters in the superpotential is zero H degenerates into the singular space $\tilde{H}$, the horizon manifold of $\tilde{K}$ given by (2.15). According to our picture, in this limit a two-cycle and a three-cycle of H shrink to zero size. We can use this fact to identify the cycles C_1 and C_2. The field theory on D3-branes at $\tilde{K}$ is $\mathcal{N}=1$ $\mathrm{SU}(N)^3$ with matter content (2.1) plus a chiral multiplet transforming in the adjoint representation of one of the gauge groups [11, 13]. In [11] it was argued that the baryon operator associated with the bifundamental field which does not couple to the adjoint, corresponds to a D3-brane wrapped on the shrinked three-cycle. We have shown that we can recover this gauge theory from our initial one by performing a double limit in which we send one of the a_i to zero and the other two to infinity. From that process we observe that if a_1 (a_2) (a_3) is sent to zero, the bifundamentals A, $\tilde{A}$ (B, $\tilde{B}$) (C, $\tilde{C}$) do not couple to the extra adjoint. Together with the assignations (3.9), this implies that C_1 is the three-cycle that shrinks to zero size when $a_3=0$, and C_2 is the one which shrinks when $a_2=0$. When $a_1=0$, the combination C_1-C_2 will shrink to zero size.

4. General case

In this section we want to determine what type of singular three-folds could lead to theories on D3-branes that generalize our previous one. In particular, we are interested in obtaining as world-volume theory an $\mathcal{N}=1$ gauge theory with group $\mathrm{SU}(N)^k$ and chiral multiplets

$$X_i = (\mathbf{N_i}, \bar{\mathbf{N}}_{\mathbf{i+1}}), \quad \tilde{X}_i = (\bar{\mathbf{N}}_{\mathbf{i}}, \mathbf{N_{i+1}}) \tag{4.1}$$

for $i=1,\ldots,k$. We will fix again the superpotential by the condition that a branch of the moduli space can be interpreted as positions of D3-branes in a transversal space.

We will follow the same steps as in section 1. We begin by considering a $\mathrm{U}(1)^k$ theory with matter content (4.1). We can construct the space of solutions to the D-term equations by using gauge invariant quantities and moding by the constraints that they are subject to [16]. A minimal set of gauge invariant quantities is

$$x_i = X_i\tilde{X}_i\,, \qquad z = X_1X_2\ldots X_k\,, \qquad w = \tilde{X}_1\tilde{X}_2\ldots\tilde{X}_k\,. \tag{4.2}$$

These quantities are subject to the relation

$$x_1x_2\ldots x_k = zw\,, \tag{4.3}$$

which defines a hypersurface in $\mathbb{C}^{k+2}$. Let us denote it by Q, $\dim_{\mathbb{C}} Q = k+1$. The moduli space of the gauge theory will be the subspace of Q determined by the F-term equations. The most general superpotential containing only quartic terms is

$$W = \sum_{i=1}^{k}(a_i x_i^2 + 2b_i x_i x_{i-1})\,. \tag{4.4}$$

The associated F-term equations imply relations expressible in terms of the variables x_i

$$b_i x_{i-1} + a_i x_i + b_{i+1} x_{i+1} = 0\,, \tag{4.5}$$

for $i = 1, \ldots, k$, with $0 \equiv k$ and $k+1 \equiv 1$. These equations define a set of hyperplanes in $\mathbb{C}^{k+2}$. If we want to interpret the moduli space of the gauge theory as positions of a D3-brane, the $2k$ parameters appearing in the superpotential cannot be all independent. They must be such that only $k-2$ of the k relations (4.5) are linearly independent. Then the moduli space of the gauge theory will be given by the intersection between Q and the $k-2$ hyperplanes associated to (4.5). We will denote again the moduli space by K; with the previous condition $\dim_{\mathbb{C}} K = 3$.

We can take the equations $i = 2, \ldots, k-1$ in (4.5) as linearly independent, and use them to express $x_3, \ldots, x_k$ in terms of x_1 and x_2. Let us define matrices A_i and B_i, of dimension $(i-2) \times (i-2)$ and $(i-3) \times (i-3)$ respectively, by

$$A_i = \begin{pmatrix} a_2 & b_3 & & & \\ b_3 & a_3 & b_4 & & \\ & & \ddots & & \\ & & & b_{i-1} & a_{i-1} \end{pmatrix}, \qquad B_i = \begin{pmatrix} a_3 & b_4 & & & \\ b_4 & a_4 & b_5 & & \\ & & \ddots & & \\ & & & b_{i-1} & a_{i-1} \end{pmatrix}.$$

In terms of A_i and B_i, and for generic values of the parameters, the variables $x_3, \ldots, x_k$ can be written as

$$x_i = (-1)^i \frac{b_2 \det B_i x_1 + \det A_i x_2}{\prod_{2<j\leq i} b_j}\,, \tag{4.6}$$

with $\det B_3 \equiv 1$. Notice that the parameters a_1, b_1 and a_k do not appear in (4.6). We can ensure that the additional two equations $i = 1, k$ in (4.5) are solved by (4.6) by setting

$$a_1 = b_2^2 \frac{\det B_k}{\det A_k}\,, \qquad b_1 = (-1)^{k+1} \frac{\prod_{1<j} b_j}{det A_k}\,, \qquad a_k = b_k^2 \frac{det A_{k-1}}{det A_k}\,. \tag{4.7}$$

Thus we are left with $2k-3$ free parameters in the superpotential. We can now substitute (4.6) in (4.3). In a generic situation, and after evident rescalings, we obtain the following defining equation for K

$$x_1 x_2 (x_1 + x_2) \prod_{i=1}^{k-3} (x_1 + \alpha_i x_2) = zw\,, \tag{4.8}$$

where $\alpha_i = \det A_i / (\det A_3 \det B_i)$. The space (4.8) is only singular at the origin, unless $\alpha_i = 0, 1, \infty$ for some i, or $\alpha_i = \alpha_j$ for some i and j.

We can easily generalize the superpotential (4.4) to that of an $\mathrm{SU}(N)^k$ gauge theory with matter content (4.1)

$$W = \sum_{i=1}^{k} \left(a_i \mathrm{Tr}\, \left(X_i \tilde{X}_i \right)^2 + 2 b_i \mathrm{Tr}\, X_i \tilde{X}_i \tilde{X}_{i-1} X_{i-1} \right), \tag{4.9}$$

where the parameters a_i and b_i are restricted as in (4.7). For the same reasons explained in section 2, this theory will have a family of vacua reproducing K^N/S_N. Notice that for particular values of the parameters a_i and b_i the space defined by (4.3) and (4.5) reproduce some of the orbifolds of the conifold treated in [13], i.e. those of the form $x^n y^m = zw$. However we are considering them as a degenerate limit of a bigger family.

The main criterion that we have used in the previous construction is that the set of solutions to the D- and F-term equations defines an space of complex dimension three. This rather naive criterion is not enough to guarantee that the resulting space K is a consistent compactification space for string theory. In order to analyze this point it is convenient to change coordinates to $x \sim x_1 + x_2$, $y \sim x_1 - x_2$. Then (4.8) can be rewritten as

$$x^k - zw + yf(x,y) = 0\,, \tag{4.10}$$

with f a polynomial function of x and y. We observe that the space K has a hyperplane section, $y = 0$, whose defining equation is that of an A_{k-1} singularity in complex dimension two. Singular three-folds having a surface of ADE singularities as a hyperplane section containing the singular point, are special cases of Gorenstein canonical singularities [1]. Therefore it is consistent to consider string theory compactified on K [11]. Using this result we propose that the low-energy theory on N D3-branes at the singular point of K, given by (4.8), is $\mathcal{N} = 1$ $\mathrm{SU}(N)^k$ with matter fields (4.1) subject to the superpotential (4.9). U(1) fields living on the world-volume of the D3-branes are expected to decouple in the infrared limit and thus we are just considering $\mathrm{SU}(N)^k$ as gauge group.

Our $\mathcal{N} = 1$ theory is related to an $\mathcal{N} = 2$ theory with the same gauge group and matter content, as it was the case for $k = 3$. Such an $\mathcal{N} = 2$ theory can be derived from D3-branes at an A_{k-1} singularity of a complex two-fold. We can break $\mathcal{N} = 2$ to $\mathcal{N} = 1$ by giving masses to the adjoint fields and integrating them out. When the mass terms are of the form $\sum m_i(\mathrm{Tr}\ \phi_i^2 - \mathrm{Tr}\ \phi_{i+1}^2)$, the superpotential of the corresponding $\mathcal{N} = 1$ theory is of the form (4.9) with parameters $a_i(m_j)$ and $b_i(m_j)$ satisfying the restriction (4.7). We observe from equation (4.10) that the space K associated to the $\mathcal{N} = 1$ theory knows about the space associated to the parent $\mathcal{N} = 2$ theory. In particular, (4.10) can be viewed as a deformation of an A_{k-1} singularity of a two-fold.[4]

There is an important difference between (4.8) and the space (2.7) associated to the case $k = 3$. While in (2.7) we can eliminate all the parameters by convenient rescalings, in (4.8) there are $k-3$ parameters that cannot be removed. Equation (4.8) describes a singularity of a three-fold whose moduli space of complex structures is

[4] We could have a priori expected that K describes an A_{k-1} isolated singularity of a three-fold. However this possibility does not seem to be consistent with the AdS/CFT correspondence, since the link sphere of an A_{k-1} singularity of a three-fold is homeomorphic to S^5 for k odd [25].

of dimension $k-3$. It is interesting to notice that the space (4.8) for $k=4$, after a linear change of coordinates, can be rewritten as

$$x^4 + ax^2y^2 + y^4 = zw\,, \tag{4.11}$$

with a a complex parameter ($a \neq \pm 2$). This is the standard form for one of the isolated unimodular singularities that a three-fold can acquire, which is denoted by X_9 in [17].

We would like to finish with a short remark on the AdS/CFT correspondence for the general case. Using the methods of [26], we can obtain the number of marginal deformations of the $\mathrm{SU}(N)^k$ theory. The number of couplings in the theory is $3k-3$, where $2k-3$ of them are parameters in the superpotential and k of them are gauge coupling constants. The vanishing of all beta functions imposes k constraints. Thus the number of marginal deformations is $2k-3$. We can consider now the supergravity metric representing N 3-branes in K. We will assume again that there exists is Ricci-flat a metric in a neighborhood of the singular point of K of the form (3.3). The decoupling of gravity limit will transform that metric into $AdS_5 \times H$, with H the horizon manifold of K. The space H inherits a moduli space of dimension $k-3$ from K. The AdS/CFT correspondence will imply that type IIB string theory on $AdS_5 \times H$ is dual to the superconformal field theory obtained by letting flow to the infrared the $\mathcal{N}=1$ $\mathrm{SU}(N)^k$ theory, with N large and $W=0$, and then perturbing it by the superpotential (4.9). From an analysis analogous to that of section 2, we obtain a consistent duality if $b_2(H) = b_3(H) = k-1$. In particular, the dual string theory would have also $2k-3$ moduli parameters: the complexified type IIB string coupling constant, the $k-3$ moduli of H and $k-1$ complex parameters coming from the integration of the B fields on the homology two-cycles.

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References

[1] M. Reid, *Young person's guide to canonical singularities*, Algebraic Geometry, Bowdoin, 1985, *Proc. Sympos. Pure Math.*, vol. **46**, part 1, Amer. Math. Soc., Providence, RI, 1987, pp. 345.

[2] J. Polchinski, *Dirichlet-branes and Ramond-Ramond charges*, *Phys. Rev. Lett.* **75** (1995) 4724 [`hep-th/9510017`].

[3] M. R. Douglas and G. Moore, *D-branes, quivers, and ALE instantons*, hep-th/9603167.

[4] M. Douglas, B. Greene and D. Morrison, *Orbifold resolution by D-branes*, *Nucl. Phys.* **B 506** (1997) 84 [hep-th/9704151].

[5] A. Lawrence, N. Nekrasov and C. Vafa, *On conformal theories in four dimensions*, *Nucl. Phys.* **B 533** (1998) 199 [hep-th/9803015].

[6] J. M. Maldacena, *The Large N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[7] S. Kachru and E. Silverstein, *4D conformal field theories and strings on orbifolds*, *Phys. Rev. Lett.* **80** (1998) 4855 [hep-th/9802183].

[8] I. R. Klebanov and E. Witten, *Superconformal field theory on threebranes at a Calabi-Yau singularity*, *Nucl. Phys.* **B 536** (1998) 199 [hep-th/9807080].

[9] S. S. Gubser, *Einstein manifolds and conformal field theories*, *Phys. Rev.* **D 59** (1999) 025006 [hep-th/9807164].

[10] S. S. Gubser and I. R. Klebanov, *Baryons and domain walls in an $N = 1$ superconformal gauge theory*, *Phys. Rev.* **D 58** (1998) 125025 [hep-th/9808075].

[11] D. R. Morrison and M. R. Plesser, *Non-spherical horizons, I*, hep-th/9810201.

[12] K. Oh and R. Tatar, *Three dimensional SCFT from M2 branes at conifold singularities*, *J. High Energy Phys.* **02** (1999) 025 [hep-th/9810244].

[13] A. M. Uranga, *Brane configurations for branes at conifolds*, hep-th/9811004.

[14] E. Witten, *Baryons and branes in anti-de-Sitter space*, *J. High Energy Phys.* **07** (1998) 006 [hep-th/9805112].

[15] S. Gubser, N. Nekrasov and S. Shatashvili, *Generalized conifolds and 4d N=1 SCFT*, hep-th/9811230.

[16] M. A. Luty and W. Taylor IV, *Varieties of vacua in classical supersymmetric gauge theories*, *Phys. Rev.* **D 53** (1996) 3399 [hep-th/9506098].

[17] V. I. Arnold, S. M. Gusein-Zade and A. N. Varchenko, *Singularities of differentiable maps*, vol.I, Birkhäuser, 1985.

[18] L. E. Ibanez, R. Rabadan and A. M. Uranga, *Anomalous $U(1)$'s in type I and type IIB $D = 4$, $N = 1$ string vacua*, *Nucl. Phys.* **B 542** (1999) 112 [hep-th/9808139].

[19] M. Reid, *Canonical 3-folds*, Journeées de Géométrie Algébriqe d'Angers (A. Beauville, ed.) Sitjhoff & Noordhoof, 1980, pp. 273.

[20] K. Hori, H. Ooguri and C. Vafa, *Non-abelian conifold transitions and $N = 4$ dualities in three dimensions*, *Nucl. Phys.* **B 504** (1997) 147 [hep-th/9705220].

JHEP02(1999)019

[21] A. Kehagias, *New type IIB vacua and their F-theory interpretation*, *Phys. Lett.* **B 435** (1998) 337 [hep-th/9805131].

[22] O. Aharony, A. Fayyazuddin and J. Maldacena, *The large N limit of $\mathcal{N} = 2,1$ field theories from threebranes in F-theory*, *J. High Energy Phys.* **07** (1998) 013 [hep-th/9806159].

[23] B. S. Acharya, J. M. Figueroa-O'Farrill, C. M. Hull, and B. Spence, *Branes at conical singularities and holography*, hep-th/9808014.

[24] N. Seiberg, *Electric-magnetic duality in supersymmetric non-abelian gauge theories*, *Nucl. Phys.* **B 435** (1995) 129 [hep-th/9411149].

[25] J. Milnor, *Singular points of complex hypersurfaces*, *Ann. of Math. Studies*, **61**.

[26] R. G. Leigh and M. J. Strassler, *Exactly marginal operators and duality in four dimensional N=1 supersymmetric gauge theory*, *Nucl. Phys.* **B 447** (1995) 95 [hep-th/9503121].

[27] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[28] E. Witten, *Anti-de-Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[29] K. Dasgupta and S. Mukhi, *Brane constructions, conifolds and M-theory*, hep-th/9811139.

[30] E. Witten, *Solutions of four-dimensional field theories via M theory*, *Nucl. Phys.* **B 500** (1997) 3 [hep-th/9703166].

JHEP02(1999)019